Oxford Cambridge and RSA

Friday 14 June 2024 – Afternoon

AS Level Further Mathematics A

Y535/01 Additional Pure Mathematics

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 **In this question you must show detailed reasoning.**

The number N is written as 28A3B in base-12 form.

Express N in decimal (base-10) form. **[2]**

2 The points A and B have position vectors **a** and **b** relative to the origin O. It is given that $\mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ 3\lambda \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} \lambda \\ -4 \\ 6 \end{pmatrix}$, where λ is a real parameter.

(a) In the case when $\lambda = 3$, determine the area of triangle OAB. **[4]**

(b) Determine the value of λ for which $\mathbf{a} \times \mathbf{b} = \mathbf{0}$. **[2]**

3 The surface S has equation $z = \mathrm{f}(x, y)$, where $\mathrm{f}(x, y) = 4x^2y - 6xy^2 - \frac{1}{12}x^4$ for all real values of x and y. You are given that S has a stationary point at the origin, O, and a second stationary point at the point $P(a, b, c)$, where $c = \mathrm{f}(a, b)$.

(a) Determine the values of a, b and c. **[6]**

(b) Throughout this part, take the values of a and b to be those found in part **(a)**.

(i) Evaluate f_x at the points $U_1\big(a-0.1, b, \mathrm{f}(a-0.1, b)\big)$ and $U_2\big(a+0.1, b, \mathrm{f}(a+0.1, b)\big)$. **[2]**

(ii) Evaluate f_y at the points $V_1\big(a, b-0.1, \mathrm{f}(a, b-0.1)\big)$ and $V_2\big(a, b+0.1, \mathrm{f}(a, b+0.1)\big)$. **[2]**

(iii) Use the answers to parts **(b)(i)** and **(b)(ii)** to sketch the portions of the sections of S, given by

- $z = \mathrm{f}(x, b)$, for $|x-a| \leqslant 0.1$,
- $z = \mathrm{f}(a, y)$, for $|y-b| \leqslant 0.1$. **[2]**

4 The first five terms of the Fibonacci sequence, $\{F_n\}$, where $n \geqslant 1$, are $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$ and $F_5 = 5$.

(a) Use the recurrence definition of the Fibonacci sequence, $F_{n+1} = F_n + F_{n-1}$, to express F_{n+4} in terms of F_n and F_{n-1}. **[2]**

(b) Hence prove by induction that F_n is a multiple of 3 when n is a multiple of 4. **[3]**

5 The set S consists of all 2×2 matrices having determinant 1 or -1. For instance, the matrices $\mathbf{P} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$, $\mathbf{Q} = -\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are elements of S. It is given that $\times_{\mathbf{M}}$ is the operation of matrix multiplication.

(a) State the identity element of S under $\times_{\mathbf{M}}$. **[1]**

The group G is generated by $\mathbf{P}$, under $\times_{\mathbf{M}}$.

(b) Determine the order of G. **[2]**

The group H is generated by $\mathbf{Q}$ and $\mathbf{R}$, also under $\times_{\mathbf{M}}$.

(c) (i) By finding each element of H, determine the order of H. **[4]**

(ii) List all the proper subgroups of H. **[3]**

(d) State whether each of the following statements is true or false. Give a reason for each of your answers.

- G is abelian
- G is cyclic
- H is abelian
- H is cyclic **[4]**

6 For positive integers n, let $\mathrm{f}(n) = 1 + 2^n + 4^n$.

(a) (i) Given that n is a multiple of 3, but **not** of 9, use the division algorithm to write down the two possible forms that n can take. **[1]**

(ii) Show that when n is a multiple of 3, but **not** of 9, $\mathrm{f}(n)$ is a multiple of 73. **[6]**

(b) Determine the value of $\mathrm{f}(n)$, modulo 73, in the case when n is a multiple of 9. **[2]**

Turn over for Question 7

7 In a long-running biochemical experiment, an initial amount of 1200 mg of an enzyme is placed into a mixture. The model for the amount of enzyme present in the mixture suggests that, at the end of each **hour**, one-eighth of the amount of enzyme that was present at the start of that hour is used up due to chemical reactions within the mixture.

To compensate for this, at the end of each **six-hour** period of time, a further 500 mg of the enzyme is added to the mixture.

(a) Let n be the number of **six-hour** periods that have elapsed since the experiment began.

Explain how the amount of enzyme, E_n mg, in the mixture is given by the recurrence system $E_0 = 1200$ and $E_{n+1} = \left(\frac{7}{8}\right)^6 E_n + 500$ for $n \geqslant 0$. **[2]**

(b) Solve the recurrence system given in part **(a)** to obtain an exact expression for E_n in terms of n. **[5]**

(c) Hence determine, in the long term, the amount of enzyme in the mixture. Give your answer correct to **3** significant figures. **[2]**

(d) **In this question you must show detailed reasoning.**

The long-running experiment is then repeated. This time a new requirement is added that the amount of enzyme present in the mixture must always be at least 500 mg.

Show that the new requirement ceases to be satisfied before 12 hours have elapsed. **[3]**

END OF QUESTION PAPER

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